



Building a stock-flow consistent model in practice

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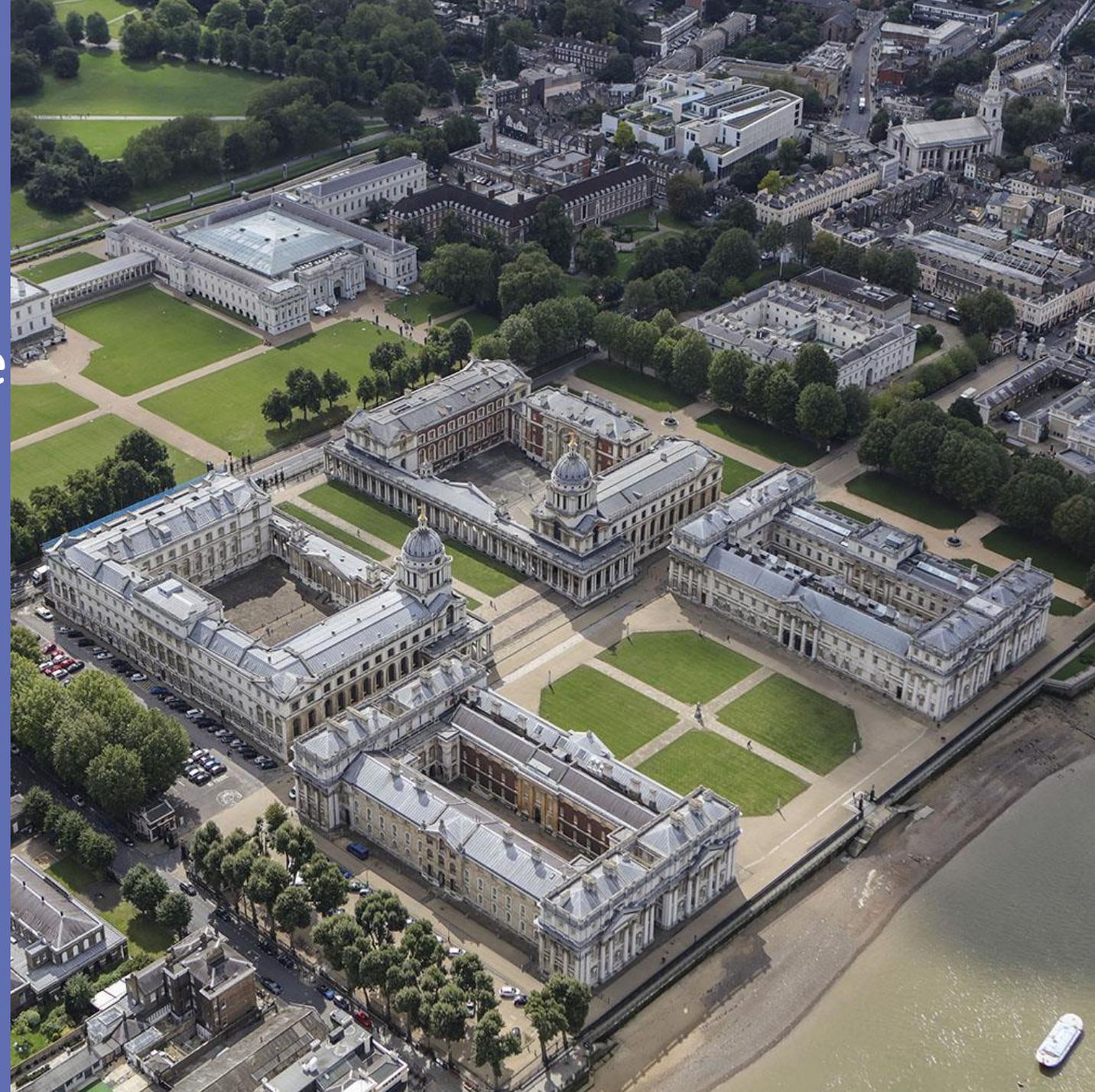
May 2018

For more information see:

<https://yannisdafemos.com/sfc-modelling/>



This event received
funding from the
European Parliament



Suppose that we have an economy with the following features:

- There are three sectors: firms, households and banks.
- **Firms** make investment by using retained profits and loans. A part of firms' profits is distributed to households.
- **Households** accumulate savings in the form of deposits.
- **Banks** provide firm loans by creating deposits. Banks' profits are distributed to households.

This is a model which includes only **private money** that is created when banks provide loans to firms.

Balance sheet matrix

	Households	Firms	Commercial banks	Total
Deposits	+M		-M	0
Loans		-L	+L	0
Capital		+K		+K
Total (net worth)	+M	+V _f	0	+K

Transactions flow matrix

	Households	Firms		Commercial banks		Total
		Current	Capital	Current	Capital	
Consumption	-C	+C				0
Investment		+I	-I			0
Wages	+W	-W				0
Firms' profits	+DP	-TP	+RP			0
Banks' profits	+BP			-BP		0
Interest on deposits	$+r_m M_1$			$-r_m M_1$		0
Interest on loans		$-r_l L_1$		$+r_l L_1$		0
Change in deposits	$-\Delta M$				$+\Delta M$	0
Change in loans			$+\Delta L$		$-\Delta L$	0
Total	0	0	0	0	0	0

Households

- Wage income of households: $W = s_w Y$ (1)
- Capital income of households: $Y_c = DP + BP + r_m M_{-1}$ (2)
- Consumption expenditures: $C = c_1 W_{-1} + c_2 Y_{c-1} + c_3 M_{-1}$ (3)
- Deposits (identity): $M = M_{-1} + W + Y_c - C$ (4)

Firms

- Output: $Y = C + I$ (5)
- Total profits of firms (identity): $TP = Y - W - r_l L_{-1}$ (6)
- Retained profits: $RP = s_f TP$ (7)
- Distributed profits (identity): $DP = TP - RP$ (8)
- Investment: $I = g_k K_{-1}$ (9)
- Capital stock: $K = K_{-1} + I$ (10)
- Loans (identity): $L = L_{-1} + I - RP$ (11)

Banks

- Profits of banks (identity): $BP=r_l L_{-1}-r_m M_{-1}$ (12)
- Deposits (redundant identity): $M_{red}=L$ (13)

Auxiliary equations

- Potential output: $Y^*=vK$ (14)
- Capacity utilisation: $u=\frac{Y}{Y^*}$ (15)
- Growth rate of output: $g_y=\frac{Y-Y_{-1}}{Y_{-1}}$ (16)
- Leverage ratio: $lev=\frac{L}{K}$ (17)

Simulating an SFC model: an example

R code

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Model equations

Households:

$$\text{Wage income of households: } W = s_w Y \quad (1)$$

$$\text{Capital income of households: } Y_c = DP + BP + r_m M_{-1} \quad (2)$$

$$\text{Consumption expenditures: } C = c_1 W_{-1} + c_2 Y_{c-1} + c_3 M_{-1} \quad (3)$$

$$\text{Deposits (identity): } M = M_{-1} + W + Y_c - C \quad (4)$$

Firms:

$$\text{Output: } Y = C + I \quad (5)$$

$$\text{Total profits of firms (identity): } TP = Y - W - r_l L_{-1} \quad (6)$$

$$\text{Retained profits: } RP = s_f TP \quad (7)$$

$$\text{Distributed profits (identity): } DP = TP - RP \quad (8)$$

$$\text{Investment: } I = g_k K_{-1} \quad (9)$$

$$\text{Capital stock: } K = K_{-1} + I \quad (10)$$

$$\text{Loans (identity): } L = L_{-1} + I - RP \quad (11)$$

Banks:

$$\text{Profits of banks (identity): } BP = r_l L_{-1} - r_m M_{-1} \quad (12)$$

$$\text{Deposits (redundant identity): } M_{red} = L \quad (13)$$

Auxiliary equations:

$$\text{Potential output: } Y^* = vK \quad (14)$$

$$\text{Capacity utilisation: } u = Y / Y^* \quad (15)$$

$$\text{Growth rate of output: } g_y = (Y - Y_{-1}) / Y_{-1} \quad (16)$$

$$\text{Leverage ratio: } lev = L / K \quad (17)$$

Model symbols

<i>BP</i>	Profits of banks	<i>M</i>	Deposits
<i>C</i>	Consumption expenditures	<i>RP</i>	Retained profits
<i>c₁</i>	Propensity to consume out of wage income	<i>r_l</i>	Interest rate on loans
<i>c₂</i>	Propensity to consume out of capital income	<i>r_m</i>	Interest rate on deposits
<i>c₃</i>	Propensity to consume out of deposits	<i>s_f</i>	Retention rate of firms
<i>DP</i>	Distributed profits	<i>s_w</i>	Wage income share
<i>g_k</i>	Growth rate of capital	<i>TP</i>	Total profits of firms
<i>g_y</i>	Growth rate of output	<i>u</i>	Capacity utilisation
<i>I</i>	Investment	<i>W</i>	Wage income of households
<i>K</i>	Capital stock	<i>Y</i>	Output
<i>L</i>	Loans	<i>Y_c</i>	Capital income of households
<i>lev</i>	Leverage ratio	<i>Y*</i>	Potential output

Steps in simulating the model in R

#Open R and create a new R script (File->New file->R script). Save this file as ‘Model’ (File->Save as).

#Clear the workspace and identify how many time periods (T) you wish your model to run. Since we use US data for the period 1960-2010, we will run the model for 51 periods. (Once you have written the commands, press ‘Source’.)

```
rm(list=ls(all=TRUE))
T<-51
```

#Download the excel file that contains the US data for the period 1960-2010 that will be used for the calibration of the model (the data come from FRED and BIS). Save the file as .csv in your desktop and insert it into R using the command below. (Once you have written the command, press ‘Source’.)

```
Data<- read.csv("//STUDENT/USR/Lab/sgsf410/Desktop/Data.csv") #this should be adjusted according to your username
```

#If you wish to estimate the mean of a variable use a command like this one (type this in Console):
mean(Data[,c("gy")])

#STEP 1: Identify the endogenous variables of the model (as well as some auxiliary variables). For each of them create a vector that has a length equal to the time periods. (Once you have written the commands, press ‘Source’.)

#Endogenous variables

```
W<- vector(length=T)
Yc<- vector(length=T)
CO<- vector(length=T)
M<- vector(length=T)
Y<- vector(length=T)
TP<- vector(length=T)
RP<- vector(length=T)
```

```

DP<- vector(length=T)
I<- vector(length=T)
K<- vector(length=T)
L<- vector(length=T)
BP<- vector(length=T)
M_red<- vector(length=T)
Y_star<- vector(length=T) #auxiliary variable
u<- vector(length=T) #auxiliary variable
gy<- vector(length=T) #auxiliary variable
lev<- vector(length=T) #auxiliary variable

```

#STEP 2: Identify the baseline scenario and select the parameter values using the table in Appendix A. In our baseline scenario here we wish our model to be at a steady state whereby economic growth is equal to the mean economic growth in the US in 1960-2010.

#Parameters

```
for (i in 1:T) {
```

```
  if (i == 1) {
```

```
    for (iterations in 1:10){
```

```

sw<-mean(Data[,c("sw")]) #sets the wage share equal to its mean value in the US during the period 1960-2010 [Category B(i)]
rm<- mean(Data[,c("rm")]) #sets the deposit interest rate equal to its mean value in the US during the period 1960-2010 [Category B(i)]
rl<- mean(Data[,c("rl")]) #sets the loan interest rate equal to its value in the US during the period 1960-2010 [Category B(i)]
c1<-0.9 #selected from a reasonable range of values [Category B(iii)]
c2<-0.75 # selected from a reasonable range of values [Category B(iii)]
u[i]<-Data[1,c("u")] #US capacity utilisation in 1960
v<-Y[i]/(K[i]*u[i]) #calibrated such that capacity utilisation in the model matches the capacity utilisation in the US in 1960 [Category C(i)]; we use equations (14) and (15)
gk<- mean(Data[,c("gy")]) #calibrated such that the model generates the baseline scenario [Category C(ii)]
c3<-(K[i]/L[i])*((Y[i]/K[i])*(1+gk)-gk-(c1*W[i]/K[i]+c2*Yc[i]/K[i])) #calibrated such that the model generates the baseline scenario; ensures that Y/K will remain constant during the simulation period [Category C(ii)]; see Appendix B
sf<-(gk-gk*(L[i]/K[i]))/(TP[i]/(K[i]/(1+gk))) #calibrated such that the model generates the baseline scenario; ensures that L/K will remain constant during the simulation period [Category C(ii)]; see Appendix B

```

#STEP 3: Select the initial values using the data for your economy or the equations of the model.

#Initial values

```

Y[i]<-Data[1,c("Y")] #US GDP in 1960 (in trillion 2009 US$)
K[i]<-Data[1,c("K")] #US capital stock in 1960 (in trillion 2009 US$)
L[i]<-Data[1,c("L")] #Loans of US non-financial corporations in 1960 (in trillion 2009 US$)

W[i]<-sw*Y[i] #derived from equation (1)
Yc[i]<-DP[i]+BP[i]+rm*(M[i]/(1+gk)) # derived from equation (2)
CO[i]<-Y[i]-I[i] #derived from equation (5)
TP[i]<-Y[i]-W[i]-rl*(L[i]/(1+gk)) #derived from equation (6)
RP[i]<-sf*TP[i] #derived from equation (7)
DP[i]<-TP[i]-RP[i] # derived from equation (8)
I[i]<-(gk/(1+gk))*K[i] #derived from equation (9)
BP[i]<-rl*(L[i]/(1+gk))-rm*(M[i]/(1+gk)) # derived from equation (12)

```

```

M[i]<-L[i] #derived from equation (13)
Y_star[i]<-v*K[i] #derived from equation (14)
lev[i]<-L[i]/K[i] #derived from equation (17)
gy[i]<-gk #based on the baseline scenario
}
}

```

#STEP 4: Write down the equations and run the model. (Once you have written the commands, press ‘Source’.)

#Equations

else {

for (iterations **in** 1:10){

#Households

```

W[i]<-sw*Y[i]
Yc[i]<-DP[i]+BP[i]+rm*M[i-1]
CO[i]<-c1*W[i-1]+c2*Yc[i-1]+c3*M[i-1]
M[i]<-M[i-1]+W[i]+Yc[i]-CO[i]

```

#Firms

```

Y[i]<-CO[i]+I[i]
TP[i]<-Y[i]-W[i]-rl*L[i-1]
RP[i]<-sf*TP[i]
DP[i]<-TP[i]-RP[i]
I[i]<-gk*K[i-1]
K[i]<-K[i-1]+I[i]
L[i]<-L[i-1]+I[i]-RP[i]

```

#Banks

```

BP[i]<-rl*L[i-1]-rm*M[i-1]
M_red[i]<-L[i]

```

#Auxiliary equations

```

Y_star[i]<-v*K[i]
u[i]<-Y[i]/Y_star[i]
gy[i]<-(Y[i]-Y[i-1])/Y[i-1]
lev[i]<-L[i]/K[i]

```

```

}
}
}

```

#STEP 5: Report your results by using tables and graphs. In the graphs that we create here we compare the actual and the simulated data. (Once you have written the commands, press ‘Source’.)

#Table

```

matrixname<-paste("Table")
assign (matrixname, (round(cbind(M_red, M, u, gy, lev, Y), digits=4)))

```

#Graphs

```

plot(Data[1:T,c("lev")], type="l", xlab= "Year", ylab= "Leverage ratio", xaxt="n")
lines(Table[1:T,c("lev")], type="l", lty=3)
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960", "1970", "1980", "1990", "2000", "2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")

```

```
plot(Data[1:T,c("u")], type="l", xlab= "Year", ylab= "Capacity utilisation", xaxt="n")
lines(Table[1:T,c("u")], type="l", lty=3)
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")
```

```
plot(Data[1:T,c("gy")], type="l", lty=1, xlab= "Year", ylab= "Growth rate of output", xaxt="n")
lines(Table[1:T,c("gy")], type="l", lty=3)
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")
```

```
plot(Data[1:T,c("Y")], type="l", lty=1, xlab= "Year", ylab= "Output", xaxt="n")
lines(Table[1:T,c("Y")], type="l", lty=3 )
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")
```

Suppose now that we wish to make one of our parameter values (the wage share) endogenous and subject to exogenous shocks based on the data. We thereby allow s_w to take the values from the data.

Replace:
`sw<-mean(Data[,c("sw")])`

with:
`sw<-(Data[,c("sw")])`

Also, replace 'sw' in the initial values and the equations with 'sw[i]'.

(Once you have done the above, press 'Source'.)

#STEP 6: Validate the model. Here we estimate only the autocorrelation for output. (Once you have written the commands, press 'Source'.)

`#install.packages("mFilter")` *#this command is necessary if mFilter has not been installed in your computer*

```
library(mFilter)
```

```
Y_log<-log((Table[,c("Y")]))
Yactual_log<-log((Data[,c("Y")]))
```

```
Y.hp <- hpfilter((Y_log), freq=100, drift=TRUE)
actualY.hp <- hpfilter((Yactual_log), freq=6.25, drift=TRUE)
```

```
acfYactual=acf(actualY.hp$cycle, lag.max=20, plot=F)
acfY=acf(Y.hp$cycle,lag.max=20, plot=F)
plot(acfYactual$acf, ylab=" ", xlab="Lag", type="l", lty=1, ylim=c(-0.5,1))
lines(acfY$acf, type="l", lty=3, ylim=c(-0.5,1))
legend("topright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")
```

#STEP 7: Re-run the simulations by changing key parameters (we skip this step here).

#STEP 8: Re-run the simulations by changing parameters that correspond to policies/institutional structures.

#First, assume that the wage share is equal to its real value in the US till 1980 and equal to 0.55 thereafter.

Create a vector for s_w :
`sw<- vector(length=T)`

Put the command below after `(i in 1:T) {`
`if (i<21){sw[i]<-Data[i,c("sw")]} else {sw[i]<-0.55}`

Delete the command:
`sw<-(Data[,c("sw")])`

(Once you have have done the above, press 'Source'.)

#Second, assume that the loan interest rate is equal to its mean value in the US till 1980, and equal to 0.25 thereafter.

Delete the following commands in order to cancel the wage share shock:
`sw<- vector(length=T)`
`if (i<21){sw[i]<-Data[i,c("sw")]} else {sw[i]<-0.55}`

Use again the following command:
`sw<-(Data[,c("sw")])`

Put the command below after `(i in 1:T) {`
`if (i<21){rl<-mean(Data[,c("rl")]} else {rl<-0.25}`

Delete the command:
`rl<- mean(Data[,c("rl")])`

(Once you have have done the above, press 'Source'.)

Appendix A: Categories of parameter values

Category	Description
(A)	Econometrically estimated parameters
(B)	Directly calibrated parameters
(Bi)	Based on data
(Bii)	Based on previous studies
(Biii)	Selected from a reasonable range of values
(C)	Indirectly calibrated parameters
(Ci)	Calibrated such that the model matches the data
(Cii)	Calibrated such that the model generates the baseline scenario

Appendix B: Estimating c_3 and s_f for the baseline scenario

In the baseline scenario we wish our economy to be at a steady state whereby economic growth is equal to the mean economic growth in the US in 1960-2010. This implies that the ratios Y/K and L/K should be constant. We calibrate the parameters c_3 and s_f in order to achieve that.

(a) Calibrating c_3 such that Y/K is constant

$$\Delta\left(\frac{Y}{K}\right) = \frac{Y}{K} - \frac{Y_{-1}}{K_{-1}} = \frac{Y}{K} - \frac{Y_{-1}(1+g_k)}{K} = \frac{\Delta Y - g_k Y_{-1}}{K} = \frac{\Delta Y}{K} - \frac{Y}{K} \frac{g_k}{(1+g_k)} \quad (\text{B1})$$

We have: $\frac{\Delta Y}{K} = \frac{C+I-Y_{-1}}{K} = \left(\frac{C}{K_{-1}} + \frac{I}{K_{-1}} - \frac{Y_{-1}}{K_{-1}} \right) \frac{K_{-1}}{K} = \left(\frac{c_1 W_{-1} + c_2 Y_{c-1} + c_3 M_{-1}}{K_{-1}} + g_k - \frac{Y_{-1}}{K_{-1}} \right) \frac{K_{-1}}{K}$ or

$$\frac{\Delta Y}{K} = \left(\frac{c_1 W_{-1}}{K_{-1}} + \frac{c_2 Y_{c-1}}{K_{-1}} + \frac{c_3 M_{-1}}{K_{-1}} + g_k - \frac{Y_{-1}}{K_{-1}} \right) \frac{K_{-1}}{K} \text{ or}$$

$$\frac{\Delta Y}{K} = \left(\frac{c_1 W(1+g_k)}{K(1+g_k)} + \frac{c_2 Y_c(1+g_k)}{K(1+g_k)} + \frac{c_3 M(1+g_k)}{K(1+g_k)} + g_k - \frac{Y(1+g_k)}{K(1+g_k)} \right) \frac{K}{K(1+g_k)} \text{ or}$$

$$\frac{\Delta Y}{K} = \left(\frac{c_1 W}{K} + \frac{c_2 Y_c}{K} + c_3 \frac{L}{K} + g_k - \frac{Y}{K} \right) \frac{1}{(1+g_k)} \quad (\text{B2})$$

Substituting (B2) into (B1), we get:

$$\Delta \left(\frac{Y}{K} \right) = \left(\frac{c_1 W}{K} + \frac{c_2 Y_c}{K} + c_3 \frac{L}{K} + g_k - \frac{Y}{K} \right) \frac{1}{(1+g_k)} - \frac{g_k Y}{K(1+g_k)} \quad (\text{B3})$$

Since Y/K should be constant at the steady state, we need $\Delta \left(\frac{Y}{K} \right) = 0$. Solving (B3) for c_3 :

$$c_3 = \frac{K}{L} \left((1+g_k) \frac{Y}{K} - g_k - \left(\frac{c_1 W}{K} + \frac{c_2 Y_c}{K} \right) \right)$$

(b) Calibrating s_f such that L/K is constant

$$\Delta \left(\frac{L}{K} \right) = \frac{L}{K} - \frac{L_{-1}}{K_{-1}} = \frac{L}{K} - \frac{L_{-1}(1+g_k)}{K} = \frac{\Delta L - g_k L_{-1}}{K} = \frac{\Delta L}{K} - \frac{L}{K} \frac{g_k}{(1+g_k)} \quad (\text{B4})$$

We have:

$$\frac{\Delta L}{K} = \frac{I - RP}{K} = \left(\frac{I}{K_{-1}} - \frac{RP}{K_{-1}} \right) \frac{K_{-1}}{K} = \left(g_k - s_f \frac{TP}{K_{-1}} \right) \frac{K}{K(1+g_k)} = \left(g_k - s_f \frac{TP(1+g_k)}{K} \right) \frac{1}{(1+g_k)} \quad (\text{B5})$$

Substituting (B5) into (B4), we get:

$$\Delta \left(\frac{L}{K} \right) = \left(g_k - s_f \frac{TP(1+g_k)}{K} \right) \frac{1}{(1+g_k)} - \frac{g_k L}{K(1+g_k)} \quad (\text{B6})$$

Since L/K should be constant at the steady state, we need $\Delta \left(\frac{L}{K} \right) = 0$. Solving (B6) for s_f :

$$s_f = \left(g_k - g_k \frac{L}{K} \right) / \frac{TP(1+g_k)}{K}$$

Appendix C: An alternative investment function

The following non-linear investment function allows you to generate endogenous cycles in the model. According to this investment function, investment is a positive function of capacity utilisation and a negative function of leverage.

$$I[i] < - (2 * (\text{mean}(\text{Data}[c("gk")])) / (1 + \exp(-10 * (u[i-1] - 0.8) + 180 * (\text{lev}[i-1] - 0.12)))) * K[i-1]$$